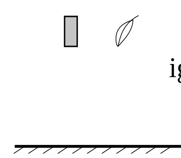
CHAPTER 5 -- NEWTON'S LAWS

QUESTION SOLUTIONS

5.1) A car moves at a constant 30 m/s. Is it accelerating? If so, must there be an applied net force to maintain this motion? Explain.

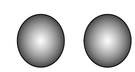
Solution: This is a tricky question. Most people see the words *constant 30 m/s* and think, *constant velocity implies no acceleration which means no force*. The problem is that velocity is a vector. There is more than one way to *change* a velocity vector. Indeed, the magnitude of the velocity vector isn't changing (it's constant), but its *direction* could be changing. Generally referred to as a *centripetal* acceleration, it is associated with a force that doesn't speed up or slow down a body but, rather, changes the *direction* of the body's motion. This requires an applied net force (N.F.L.--objects in motion tend to stay in motion in a straight line *unless impinged upon by an external force*). In short, *the car may or may not be accelerating. If it is moving along a curved path, there must be a net force motivating that motion.*

5.2) Two objects of different mass will have different weights (that is, they will feel different gravitational forces), yet if air friction is ignored and you drop both from the same height, they will accelerate due to gravity at the same rate. How can this be?



Solution: Due to a quirk in the nature of things, if a block has twice the gravitational mass (i.e., twice the willingness to be attracted to another body . . . like the earth) as a feather, it will also have twice the inertial mass (i.e., twice the resistance to changing its motion) as does the feather. The greater weight (gravitational force) is counteracted by the body's greater resistance to changing its motion (inertia), and the net effect is that all objects in a frictionless situation will fall at the same rate.

5.3) A solid copper ball and a hollow copper ball of the same radius are found in space. Both are weightless. Without cutting them open, how can you determine which is which?



Solution: Because the two objects are both made of copper, the hollow ball will be less massive. Being less massive, it will have less resistance to changing its motion. Therefore, if you are in space where the only force applied to either ball comes from you, the ball that is easier to motivate out of a stationary state will be the hollow, less massive ball. (This is like asking, *What will be harder to move in space, a 16 pound shot-put or a softball-size sphere made of styrofoam?*)

5.4) A body is accelerated by some net force. If the force is halved, how will the velocity-change Δv alter? If the mass is halved instead, how will the velocity-change Δv alter?

Solution: *Velocity change* with time is acceleration, so *altering* the velocity change (as the question states) really suggests that there is a change of acceleration. According to Newton's Second Law, net force and acceleration are proportional ($F_{net} = ma$), so if the force is halved with the mass kept constant, the acceleration will also be halved. On the other hand, if the force is kept constant and the mass is halved, the acceleration must double (again, think about $F_{net} = ma$).

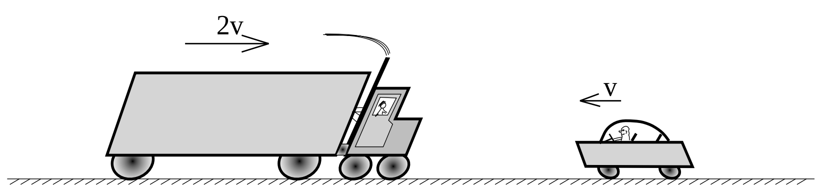
5.5) It is easier to keep a crate moving across a frictional floor than it is to get it going in the first place. Why? That is, aside from the relative motion, what is the fundamental difference between the two cases?

Solution: You are looking at the difference between kinetic (sliding) friction and static friction. Static friction, due to the fact that there is no motion involved, is a situation in which the atomic structures of the two interacting bodies can meld more completely than would be the case with sliding friction. As that melding must be sheared to instigate motion, it is easier to keep a crate going (less melding, hence, less shearing needed) than to get it going in the first place.

5.6) A heavy box attached to a parachute will reach the ground faster than a light box of equal size attached to an identical parachute. Why?

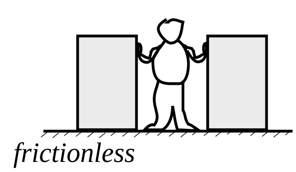
Solution: As a parachute falls, air collides with the underside of the plummetting chute. This produces an upward, resistive force on the chute. With essentially no initial velocity at first, the dominating force in freefall is gravity. But as the velocity increases, this air-born resistive force increases (remember, it's opposite the direction of gravity) diminishing the net force on the chute and reducing its acceleration. When the force of gravity exactly counteracts this resistive force, the body stops accelerating and proceeds to the ground at what is called *terminal velocity*. To reach terminal velocity, a heavy box must be moving quite fast before air friction can counteract its relatively large weight. As such, a heavier box will have a higher terminal velocity associated with it and, as a consequence, will reach the ground faster than will a lighter box.

5.7) A truck is ten times more massive and moves with twice the speed of a small car. The two collide. During the collision, which will experience the greater force? Which will experience the greater acceleration?



Solution: This is an old standby. According to Newton's Third Law, for every force in the universe there must be an equal and opposite *reaction* force. (Minor point: The word *reaction* is misleading as, in fact, the two forces happen at the same time . . . versus one happening and the other following in reaction). The idea is that one object can't exert a force on another object without the other object exerting the same force back on the first. It's just the way the universe works. What this means is that whatever force magnitude the truck experiences, the car must experience the same force magnitude. An object's acceleration is more complicated as it is related to both the magnitude of the net force (again, this is the same for both in this case) *and* the body's mass (remember, $F_{net} = ma$). In this case, the car (with its lesser mass) will experience a greater acceleration(i.e., it will change velocity more radically) than will the more massive truck with its lesser acceleration . . . even though the product ma will be the same for both.

5.8) A man stands wedged between two identical crates on a frictionless sheet of ice. Is there any way he can make the acceleration of one of the blocks greater than that of the other?



Solution: If the man pushes one block, his interaction with that block will force him toward the other block. That is, if he pushes the right block to the right, it will push on him to the left with an equal force. Given the fact that he is jammed between the two boxes, that force will be transferred through him to the block on the left. As the two blocks have the same mass, the equal forces being applied to each will provide the same acceleration no matter what.

5.9) Drop a rock from the mast of a moving boat. Will it hit the deck a.) in front of the mast, b.) next to the mast, or c.) behind the mast? Justify your response in terms of the force(s) acting on the rock.

Solution: The only force acting on the rock, ignoring frictional effects (we can effectively do this because the relative distance traveled is so small), is gravity. If the rock has an initial velocity in the horizontal (it does as it is initially traveling with the boat), it will continue to move with that horizontal velocity until it hits the deck. As the horizontal velocity of the rock and mast will be the same throughout, the rock should hit the deck next to the mast . . . and *b* is the answer.

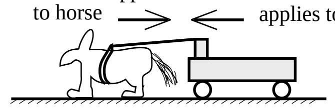
5.10) A kid steps off a footstool and begins to freefall under the influence of gravity. That is, the earth applies a gravitational force to the kid. For this situation, what is the "reaction force" alluded to in Newton's Third Law?

Solution: In the first place, the term *reaction force* is a misnomer. The "action" and "reaction" forces really act at the same time. Ignoring this unfortunate play on words, if the earth exerts a gravitational force on the kid, the kid must exert an EQUAL AND OPPOSITE gravitational force on the earth. The reason the kid's response to the force is so noticeable (after all, the little devil *does* seem to pick up speed fairly quickly as he/she moves toward the earth) is because the child's mass is small in comparison to that of the earth. Remember, $F_{net} = ma$. For a given force, if m is small, a will be relatively large . . . and vice versa.

5.11) A force F_1 stops a car. In terms of F_1, how large must a new force be to stop the same car under the same circumstance but in half the distance? In half the time?

Solution: This is tricky. It seems to make sense that to stop the car in half the time, the car has to slow down twice as fast. In fact, the relationship between acceleration and time is more easily seen by considering a car at rest that accelerates through some distance. If you decide to cover the distance in half the time, how does the acceleration have to change? The relationship $\Delta x = .5at^2$ suggests that travel distance is dependent upon the square of the time (yes, this is for a somewhat different problem--a car picking up speed instead of one slowing down--but the concept is the same). In other words, halving the time requires the acceleration to increase by a factor of *four* if the distance is to stay the same. Given the fact that acceleration is proportional to force, the force in this case must be $F = 4F_1$. On the other hand, stopping the car in half the distance really does mean that the car has to slow down twice as fast as it originally did. The math justifies this claim. Specifically, $v_{stop}^2 = v_o^2 + 2a\Delta x$ with $v_{stop} = 0$ and a being negative means that $a = v_o^2/2\Delta x$ (note that the sign of the acceleration term has been unembedded leaving a to denote a magnitude only). Therefore, if Δx halves, a doubles and the force we are looking for becomes $F = 2F_1$.

5.12) A horse pulls on a cart. According to Newton's Third Law, the cart must exert an equal and opposite reaction force back on the horse. That is, if the horse pulls the cart with 50 newtons of force, the cart must pull back on the horse with 50 newtons of force. As this so-called action/reaction pair always adds to zero, it appears as though we are suggesting that the cart will never accelerate. This obviously can't be the case, so what's the problem here?



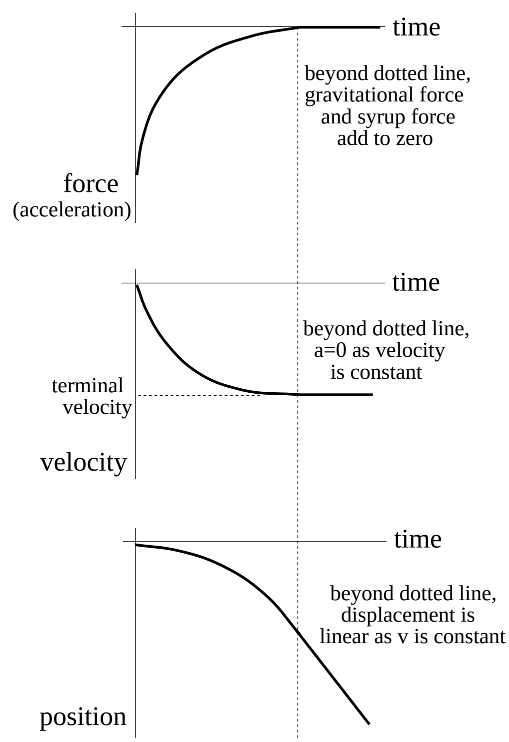
(what's the matter, don't recognize a horse when you see one?)

Solution: I first saw this problem in Hewitt's book *Conceptual Physics*. It is a great example of getting Newton's Laws confused. There is, indeed, an action/reaction pairing between the horse and the cart, but that isn't all there is. For motion, there must be a force between the horse's hooves and the street (in what direction is that force? . . . it had better be in the direction the horse is trying to go--if this isn't clear, ask your teacher about it). In other words, if you are going to look at the forces acting on a system, you have to include *all* the forces.

5.13) Assuming friction is negligible, which will reach the bottom of an incline first, a large box or a small box? Explain.

Solution: In theory, if the incline is frictionless, both would hit the bottom at the same time. For a frictional situation, though, with the big box weighing more, kinetic friction will be greater on that box than on the smaller box so the smaller box should get to the bottom first.

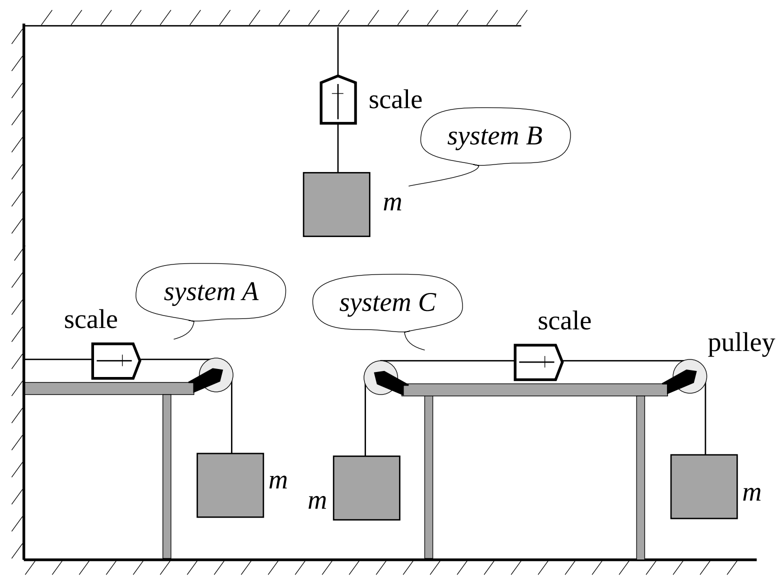
5.14) An object falls from rest into a syrupy fluid. What does its *net force versus time* graph look like? Its *velocity versus time* graph? Its *position versus time* graph?



Solution: The *force versus time* graph will look the same as the *acceleration versus time* graph. Knowing that, we can determine the general form of the *velocity versus time* graph and *position versus time* graph using what we know about the relationship between those quantities. Note that if the object had fallen in air, it would have begun with an acceleration of 9.8 m/s^2 only to have its acceleration decrease as its velocity and, hence, frictional effects increased. At some point, the frictional force would exactly match gravity, the acceleration would cease, and the object would move at a constant *terminal velocity* for the rest of its trip. In a syrupy fluid, the same will happen except it will hit terminal velocity more quickly. The graphs are to the right. Note that the *force (acceleration) versus time* graph is just the derivative (i.e., slope) of the *velocity versus time* graph which is, in turn, the slope of the *position versus time* graph. Assuming POSITIVE is associated with UPWARD MOTION, everything is negative.

Note: In doing this solution, I started with the easiest motion to visualize, at least for me. That was the *velocity versus time* graph. I figured the velocity started from rest, increased at some non-linear rate until it leveled off at terminal velocity, then stayed there for the rest of the motion. From that graph, I deduced the other two graphs.

5.15) Assuming all masses are the same size and the pulleys are ideal (i.e., massless and frictionless), which of the three scales in the figure will register the greatest force?



Solution: As bizarre as this might seem, they will all register the same force. The scale in the single pulley system (system A) on the left is essentially registering the weight of *m* (all an ideal pulley does is re-direct the line of the tension), so it and the ceiling hung scale (system B) will read the same number. The one that usually confuses people is the double pulley situation (system C) on the right. Reconsider, though, system A. To keep the hanging mass from accelerating downward, the wall must apply a force that translates to the scale and mass via tension. That force pulls the scale to the left. Now consider system C. Its right side looks exactly like the system A (that is, there is a scale, a pulley, and a hanging mass). For the right side of system C to be in equilibrium, there must be a force applied to its scale *that is the same size* as the force applied to the scale in system A. Where does that force come from in system C? It comes from the hanging mass on the left side of the table. What's important to realize is that because the two situations both depict equilibrium, the force on the left side of the scale in both cases must be the same. And if that is so, both scales must measure the same value. Put a little differently, it doesn't matter how the force required for equilibrium is generated, if the hanging mass on the right side of each table is to sit in equilibrium, the same force must be applied to the left side of the scale *no matter where that force comes from*. In the one case, the wall provides the force. In the other case, a second hanging mass provides the force. In all cases, though, the scales will read the same value.

5.16) A ball is dropped from rest a distance *h* units above a bathroom scale. When it hits, the scale measures an average force of 50 newtons. The ball is then dropped from a distance *2h* units above the scale. Will the scale read 100 newtons, less than 100 newtons, or more than 100 newtons? Explain.

Solution: It turns out that it will read *less than 100 newtons*. The key here is in determining the velocity of the ball as it strikes the scale so that we can determine the acceleration that must exist if the ball is to come to rest (remember, force and acceleration are proportional). If we assume the time of collision is the same for both situations (a dubious assumption, but we have little else to go on here so we have to make it), and if we assume the acceleration is approximately constant throughout the slowdown (another dubious assumption, but what the hell), we can use kinematics to write $F_{avg} = ma_{avg} = m(v_d - v_s)/t_{collision}$, where v_d is the velocity of the ball when the scale is fully depressed (this value will be *zero* at that point) and v_s is the velocity of the ball just as it hits the scale. With $v_d = 0$, this becomes $F_{avg} = -mv_s/t$ (in this case, we can ignore the negative sign--we are only interested in the magnitude of the force). Also, we know that this number will numerically equal 50 newtons when the ball falls from a height h, so we can re-write our expression (ignoring the negative sign) as $50 = mv_s/t$. If we can determine a relationship between the *before collision* velocity v_s for both cases (after all, according to our expression, v_s is proportional to the average force), we can determine the relationship between the average force generated by the two falls. For the fall of height h, this expression is $v_s^2 = v_o^2 + 2(-g)(-h)$, with v_o being the initial fall velocity (this is zero). That relationship yields $v_s = (2gh)^{1/2}$. Using this same expression for a height of $2h$ yields $v_{2h} = (2g(2h))^{1/2} = (2)^{1/2}v_s$. In other words, $v_{2h} = (2)^{1/2}v_s$. As the forces are proportional to these velocities, we can write $F_{avg,2h} = (2)^{1/2}F_{avg,h} = (2)^{1/2}(50\ nts) = 70\ nts$. The new force will be less than 100 newtons.

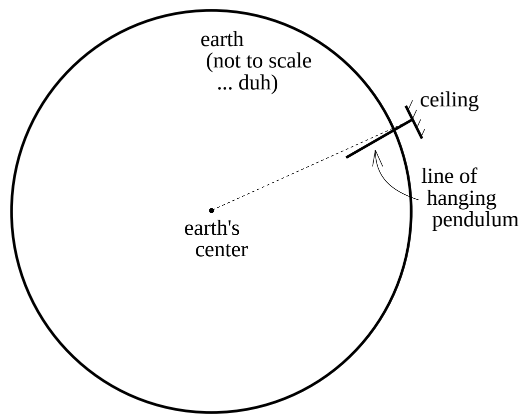
5.17) A massive object is placed on a frictionless table. It takes 2 newtons of force to accelerate it at .5 m/s^2. The object is taken into space where it is weightless. The force required to accelerate the object at .5 m/s^2 will be (a) less than, (b) equal to, or (c) more than 2 newtons.

Solution: On a frictionless table, all your force is doing as it accelerates the object is overcoming the object's inertia. In gravitationless space, the same is the case. The two forces should, therefore, be the same.

5.18) Magic Mountain is an amusement park in Southern California that is known for its giant roller coaster rides. One of the rides, Superman, consists of a cart that is accelerated along a horizontal stretch of track (via magnetic induction, no less) to somewhere around 100 mph in less than four seconds. The cart's path then curves into a vertical climb up an enormously high tower. At the top it comes to rest whereupon it proceeds to freefall several hundred feet back down the vertical section of the track and out the curve onto the horizontal section where it finally comes to rest. The drop from the top is billed as *pure freefall*. In theory, if you took this ride and released a dime at the top (i.e., just as you began to freefall back down the track), the dime should sit motionless in front of you as both you and it gravitationally accelerated back toward Earth. For two reasons (one obvious and one not so obvious), DOING THIS WOULD BE A BAD IDEA--A REALLY, REALLY BAD IDEA. What is the obvious problem and, for the hotshots, what is the not so obvious problem?

Solution: If you go along with the idea that you don't want to kill someone, then you don't want to do anything that might allow the freely falling dime to come into contact with someone's skull. If you and the dime were accelerating at the same rate, it would still be in front of you when you got to the bottom and you could grab it before pulling out onto the horizontal stretch. The "obvious problem" is that that won't happen. The dime, being relatively light, will hit its terminal velocity fairly quickly and, as a consequence, will not keep up with you as you accelerate downward (it will appear to accelerate up and away from you). If people happen to be walking under the ride, they could get clobbered by the falling object (amusement parks get around this problem by closing off the area under such rides). The "not so obvious problem" is associated with what is called Bernoulli's effect. As the dime falls, the air velocity on one side of the dime will be greater than on the other side due to its spin. As a consequence, the variation will cause a pressure difference between the two sides. The high pressure side will push the dime motivating it to veer off away from you to the left or the right. If conditions are just right (or just wrong, depending upon how you look at it), the dime could conceivably sail out beyond the restricted area under the ride and actually hit someone. In short, if you must observe this freefall phenomenon, use something relatively massive (maybe a baseball?) whose terminal velocity won't be reached so quickly and whose mass won't allow Bernoulli's effect to push it around so easily.

5.19) A pendulum in Los Angeles (22^o latitude) does not hang directly toward the center of the earth. Explain why not.



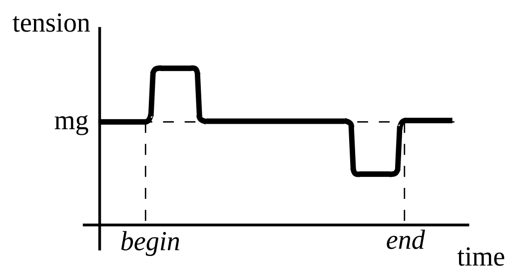
Solution: A simple pendulum is a string with a mass attached to one end. Think about holding such a pendulum while standing on the edge of a rotating *merry go round*. From your perspective, what does the pendulum bob appear to do? It seems to push out away from the center (actually, it's really trying to follow straight-line motion--your holding it, via the string, applies a force that pulls it into circular motion . . . hence the feeling that it pushes you outward). If the earth were stationary, a pendulum would be gravitationally attracted to the center of the earth and the string would orient itself between the pendulum's contact point (i.e., where the pendulum is attached to, say, the ceiling) and the earth's center. The problem is that the earth is rotating. This means that along with the tension force required to counteract gravity, there must be a tension *component* that pulls the pendulum bob into circular motion. The consequence is that the line of the pendulum will not be toward the earth's center but will be off a bit toward the equator (see sketch).

5.20) A block on a frictional incline plane compresses an ideal spring by a distance d. The spring is released firing the block up the incline. Old George maintains that the block will go up the incline and, upon returning, will recompress the spring by some distance *less than d*. Why, in the real world, might he think that, and what additional, exotic thing might happen that could prove him wrong?

Solution: If the incline had been frictionless, the magnitude of the block's velocity as it left the spring would, due to the symmetry of the situation, have been the same as the magnitude of the velocity when it returned. If that had been the case, one would expect that the spring would again be depressed the same distance d. With friction in the picture, the block would not rise as high on the incline as otherwise would have been the case, and would not be moving as fast when it got back down to the spring. In that situation, you would expect the spring to be depressed a distance *less than d*. The exotic twist comes in the fact that *kinetic frictional force* is not as strong as *static frictional force*, so it is possible that when the block gets to the top of its motion, the gravitational component will not be large enough to overcome the *static frictional force* being applied and the block will simply stay there without returning down the incline at all. Tricky, eh?

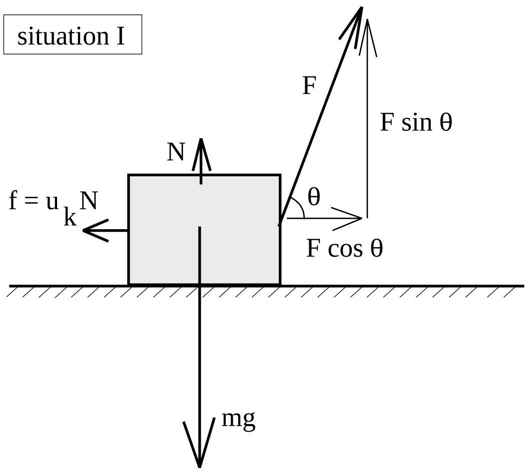
5.21) A mass *m* is attached to one end of a string. The other end of the string is attached to the ceiling of an elevator. The elevator proceeds from the first floor to the sixtieth floor. What might you expect the graph of the string tension to look like, relative to the force *mg*, as the motion proceeds? Being the big-hearted guy that I am, I'll give you a hint: the tension IS *mg* before the elevator begins to move. Justify each part of your graph.

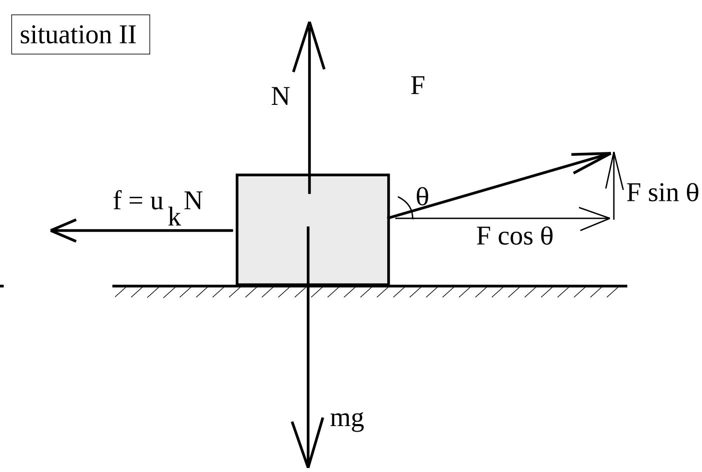
Solution: The tension in a string hanging from a ceiling will equal the weight of the mass attached to it as long as the system isn't accelerating. If there is acceleration, the tension will be other than *mg*. So, the elevator accelerates upward at a constant rate (I'm assuming it's constant) to start. The tension must not only support the weight of the pendulum bob, it must also increase the bob's velocity. In other words, the tension associated with that initial acceleration will be greater than *mg*. When the elevator reaches cruising speed, assuming it doesn't accelerate throughout the ascent, the velocity *change* will be zero and, hence, so will the acceleration. In that case (i.e., for constant velocity), the tension in the line will simply be *mg* (note that this is a consequence of N.F.L.: objects in motion stay in motion with a constant velocity *unless impinged upon by a net external force*--in this case, gravity and tension are external forces that add to zero so the net force on the bob is zero and the bob proceeds with constant velocity). When the elevator approaches the end, it must accelerate negatively, slowing the body down (if you've ever been in an elevator that is doing this, you know that you end up feeling light on your feet--the same happens here). The tension doesn't have to support the full weight of the pendulum bob because the bob's velocity is decreasing (it's the opposite of having to increase the tension to counter both gravity *and an increase* in velocity). The graph depicts all of this.



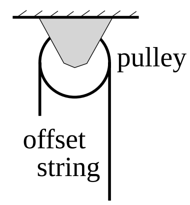
5.22) A block on a horizontal frictional surface is pulled by a rope oriented at some non-zero angle, relative to the horizontal. Is there an optimal angle at which the block's acceleration will be a maximum and, if so, how would you go about theoretically determining that angle?

Solution: There are two forces to deal with here. The easier one to see is the force generated by the rope (I'll call this *F*). It will have two components, one in the vertical and one in the horizontal. The second is friction. The acceleration of the block will depend upon the vector sum (actually, because they are in opposite directions, it'll be a subtraction) of the frictional force and the horizontal component of *F*. The frictional force depends upon the *normal force* which, in turn, depends upon the *vertical component* of *F*. The relationships can be seen in the sketches labeled *situation I* and *situation II*.





It should be obvious from looking at the sketches that with a big angle comes a little normal force (the large vertical component of F means that the surface providing the normal force will not have to muster much force to counteract mg), a small frictional force (N will be small, so the coefficient of kinetic friction times N will be small), and a small horizontal component of F. The *vector* sum of friction and the horizontal component of F (this will be $f - F\cos\theta$) will be small as the two are about the same size. As a consequence, the acceleration will be relatively small. Likewise, with a small angle comes a large normal force, a large frictional force, and a large horizontal component of F. The *vector* sum of friction and the horizontal component of F will, again, be small as the two will be about the same size. As a consequence, the acceleration will be relatively small. There is, indeed, an angle somewhere between the two situations in which the normal force isn't too big so the frictional force isn't too big, but the horizontal component of F is largish. In that case, the vector sum of the horizontal vectors will be relatively large as will the acceleration. How do you get that angle? Use N.S.L. to generate an expression for F as a function of θ, then maximize that function (i.e., note that the slope of the F *vs* θ graph will be zero at a maximum, take the derivative of F to determine its slope function, set that function equal to zero to hone in on its maximum, then solve for the angle that satisfies that situation).



5.23) You want to set up the following device: Place a string over a pulley, then attach unequal masses to each end (this device is called *an Atwood Machine*--typically, the question asked for such a device is *what will the acceleration of the system be if allowed to freefall?*). Unfortunately, you are symmetrically challenged. Every time you thread the string over the pulley (i.e., before you get the masses attached), you put more string on one side of the pulley than on the other side. That means that when you release the string to pick up the masses, the string free-wheels over the pulley and ends up on the ground (really irritating).

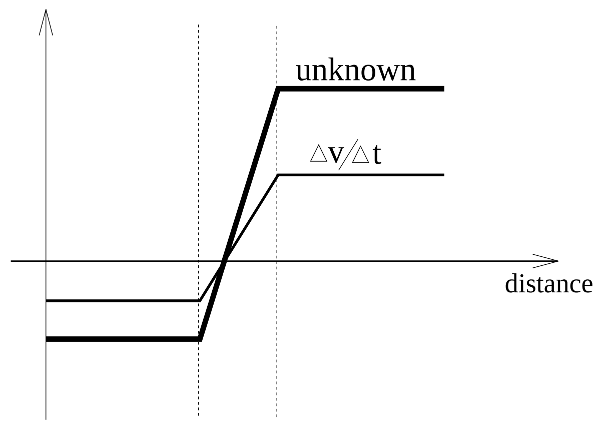
You may be symmetrically challenged, but you aren't stupid. Realizing that you aren't going to be able to make the miserable thing work, you change the problem to *what is the* *string's* *freefall acceleration as it free-wheels over the pulley*? Without answering the question itself, answer the following: Will the acceleration be constant (i.e., could you use kinematics on this if you were clever?)

and, if not, what parameters (i.e., height above the ground, initial velocity, what?) will determine what the string's acceleration is at a given instant?

Solution: What determines the acceleration will be the disparity in weight between the string on one side of the pulley and that on the other side. As that disparity will change with time (i.e., as more string slides over the pulley, more weight will be on the down side), the acceleration would not be constant and kinematics would not be an acceptable option.

5.24) A net force of 2 newtons is applied to a mass. The speed of the object doesn't change. How can this be?

Solution: If the force is perpendicular to the direction of motion, the magnitude of the velocity won't change but the direction will. Evidently, that is what is happening here.

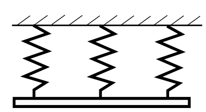


5.25) The graph shown depicts an object's change of velocity with time. What might the unknown function be?

Solution: *Change of velocity with time* is acceleration, so the known function mimics that of acceleration. According to N.S.L., $F_{net} = ma$. As the known and unknown functions appear to be proportional to one another, I suspect the unknown function is that of the net force on the object.

5.26) John was big, but he wasn't too bright. He needed to transport several 60 pound cubical microwave cookers (each was enameled plastic with no feet) across town in his car, but his beat up, rusty hulk of a vehicle was just a little too small. To accommodate the last two, he put them side by side on the roof. Once there, he urged the car forward only to find that they both broke loose when stopping. To remedy the problem, he put one on top of the other thereby doubling the normal force thereby doubling the frictional force. What problem is he likely to run into?

Solution: He is only doubling the normal force on the bottom microwave. Assuming the car's surface isn't slick (it *was* rusty), the top one will not enjoy the extra friction.



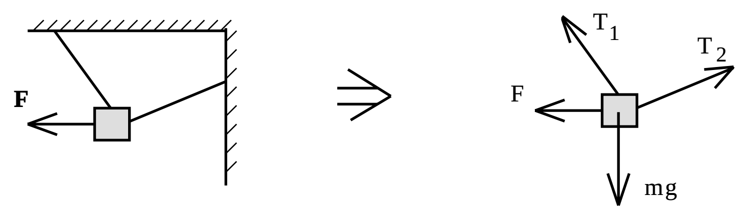
5.27) Three identical springs are attached at the ceiling. A bar of mass m is hooked to the group. If the new system's equilibrium position is d units below the springs' unstretched lengths, what must the spring constant be for each spring?

Solution: Summing the forces at the new equilibrium position yields 3kd - mg = ma = 0 (i.e., a = 0 at equilibrium). Solving yields a spring constant of k = mg/(3d).

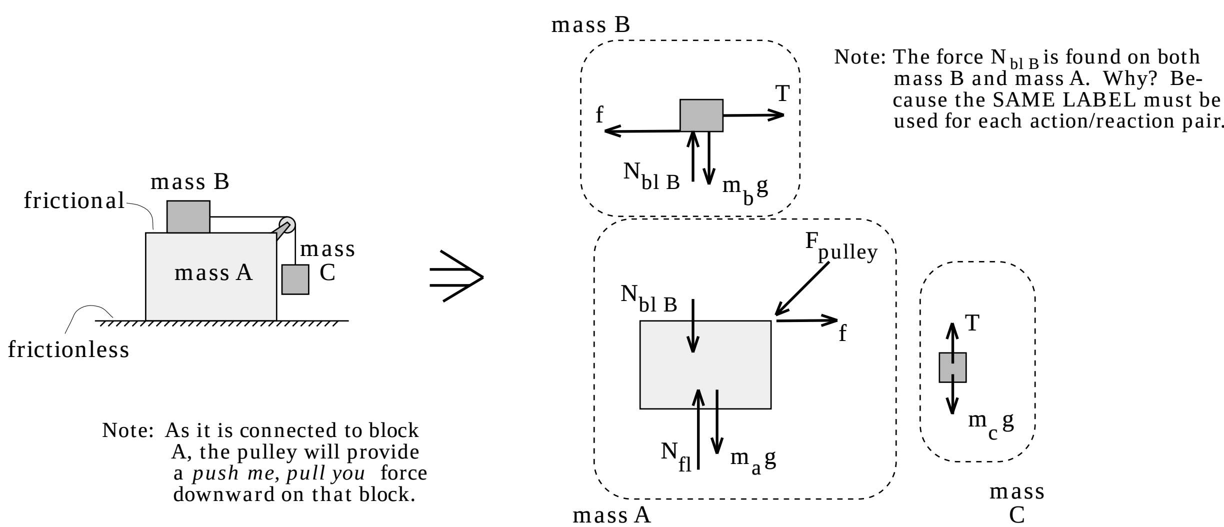
PROBLEM SOLUTIONS

5.28) Drawing a *free body diagram* for the force of EACH BODY in each sketch:

a.)



b.)

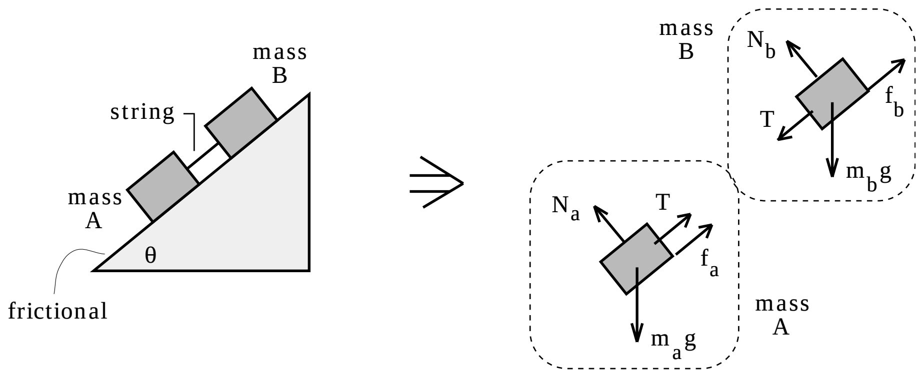


Note 1: There are two *action/reaction force pairs* between *masses A* and *B*: the normal force $N_{bl\ B}$ that A applies to B and vice versa, and the frictional force f between the two. Be sure you understand what is going on here!

Note 2: Notice that the *magnitude* of the tension force T on *mass C* and *mass B* is the same.

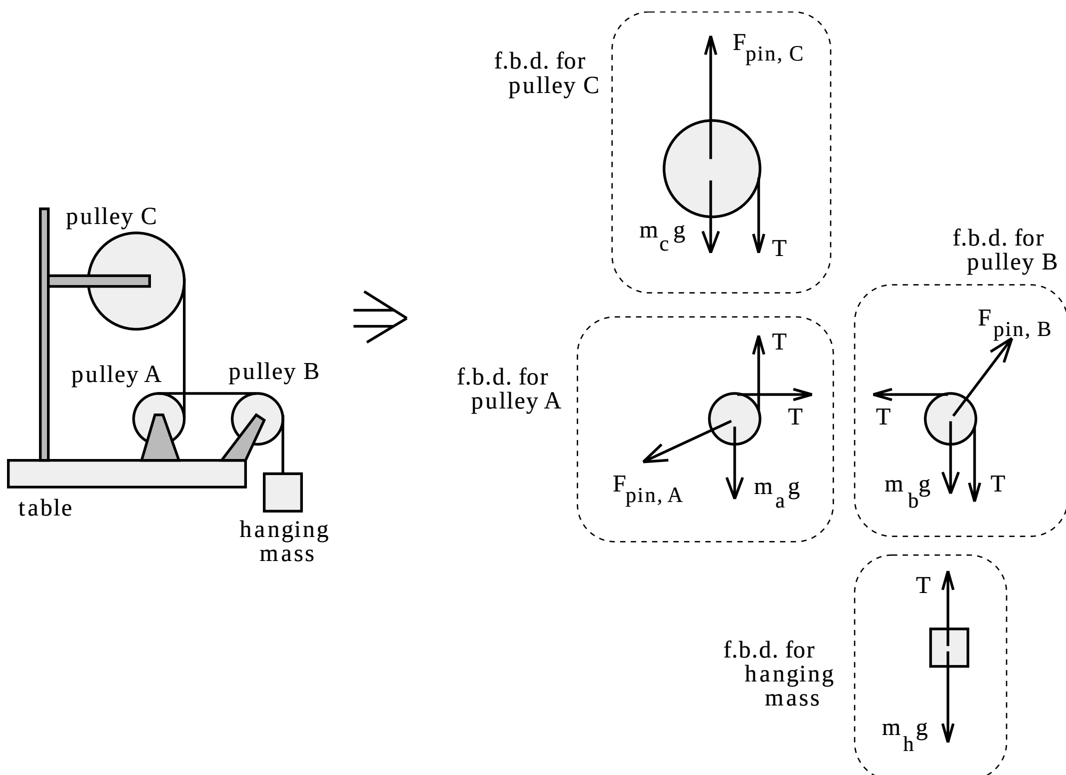
Note 3: The pulley mount on *mass A* applies a *downward and to the left* force F_{pulley} on *mass A*. As we are interested in ALL the forces acting on each mass, that force has to be included.

c.)



d.)
Note 1: All the pulleys do here is re-direct the line of the tension *T*.

Note 2: The pin that holds each pulley in place must exert a force that effectively keeps its pulley from flying off into space.



Note 3: There is a force acting at *the pin of each pulley* to keep the pulleys from falling through the table.

5.29) According to Newton's Third Law:

a.) The reaction to *the force the floor applies to you* is *the force you apply to the floor*.

b.) The reaction to *the force a string applies to a weight* is *the force the weight applies to the string*.

c.) The reaction to *the force a car applies to a tree* is *the force the tree applies to the car*.

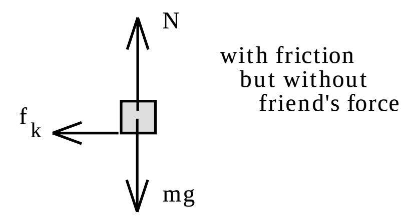
d.) The reaction to *the force the earth applies to the moon* is *the force the moon applies to the earth*.

5.30)

a.) A *free body diagram* for the situation *before your friend applies his force* (*Part B*) is shown below. Making a_x into a MAGNITUDE by unembedding the negative sign, N.S.L. yields:

$\sum F_x$:

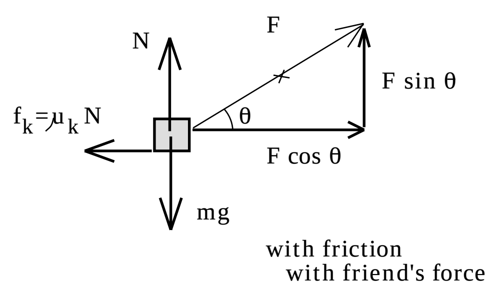
$$- f_k = - ma_x$$
$$\Rightarrow \quad - (12 \text{ nt}) = - (30 \text{ kg})\, a$$
$$\Rightarrow \quad a = .4 \text{ m/s}^2.$$



Note 1: Why make a_x into a magnitude by unembedding the negative sign? In certain kinds of problems, doing so will make life easier. Get used to it.

Note 2: In the next question, you are going to need μ_k. From the f.b.d. above, $N = mg = (30 \text{ kg})(9.8 \text{ m/s}^2) = 294 \text{ nts}$. As $f_k = \mu_k N$, we can write $\mu_k = f_k/N = (12 \text{ nt})/(294 \text{ nt}) = .04$.

b.) With the additional force applied by your friend, the *free body diagram* looks like the one shown to the right (note that *N* has changed). To determine *N*:



$\sum F_y$:

$$N + F \sin 40^\circ - mg = - ma_y \qquad (= 0 \text{ as } a_y = 0)$$
$$\Rightarrow \quad N = -F \sin 40^\circ + mg$$
$$= - (60 \text{ nt}) \sin 40^\circ + (30 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 255 \text{ nts}.$$

$\underline{\sum F_x}$:

$$- \mu_k N + F \cos 40^o = - ma_x.$$

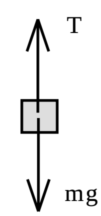
Note 1: In this case, I have assumed that your friend's force will not overcome that of friction and the *direction* of the sled's *acceleration* will still be negative (i.e., to the left). As such, I have unembedded the negative sign in front of the *ma* term. If I am wrong, the SIGN of the calculated acceleration will be negative. Continuing:

$$- \mu_k N + F \cos 40^o = - ma_x$$
$$\Rightarrow \ - (.04)(255 \text{ nt}) + (60 \text{ nt})(.766) = - (30 \text{ kg})\, a$$
$$\Rightarrow \qquad a = -1.19 \text{ m/s}^2.$$

Note 2: The negative sign means that I've assumed the wrong direction for *a*. Evidently, your friend's force was greater than the frictional force and the acceleration was really in the *+x direction* (if this ever happens to you, what I've just said is all you will have to state to make the problem OK).

5.31)

a.) A stationary elevator will feel no friction; the f.b.d. for the situation is shown in the sketch to the right. Using N.S.L.:

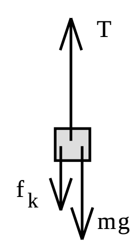


$\underline{\sum F_y}$:

$$T - mg = ma$$
$$= 0 \qquad \text{(as elevator's acc. } a_e = 0\text{)}$$

$$\Rightarrow \ T = mg$$
$$= (400 \text{ kg})(9.8 \text{m/s}^2)$$
$$= 3920 \text{ nts.}$$

b.) With the upward acceleration of the elevator, the frictional force will be applied downward as shown in the f.b.d. to the right. The acceleration term *a* is a magnitude whose sign (manually placed) is positive. N.S.L. yields:

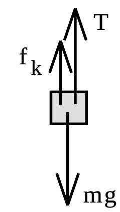


$\underline{\sum F_y}$:

$$T - mg - f_k = +ma$$
$$\Rightarrow \ T = mg + f_k + ma$$

$= (400\text{ kg})(9.8\text{m/s}^2)+(80\text{ nt})+(400\text{ kg})(2.8\text{ m/s}^2)$
$= 5120\text{ nts.}$

c.) The only difference between this problem and *Part b* is that the acceleration is zero (constant velocity means zero acceleration). It makes no difference what the velocity actually is; the forces acting on the elevator are the same as in *Part b* so the f.b.d. from *Part b* is still valid. Using it, we get:



$\underline{\sum F_y}$:

$$\begin{aligned} T - mg - f_k &= ma \\ \Rightarrow\quad T &= mg + f_k + m(0) \\ \Rightarrow\quad &= (400\text{ kg})(9.8\text{m/s}^2) + (80\text{ nt}) \\ &= 4000\text{ nts.} \end{aligned}$$

d.) With the downward velocity, friction is upward as shown in the f.b.d. to the right. N.S.L. yields:

$\underline{\sum F_y}$:

$$\begin{aligned} T - mg + f_k &= -ma \\ \Rightarrow\quad T &= mg - f_k - ma \\ &= (400\text{ kg})(9.8\text{m/s}^2) - (80\text{ nt}) - (400\text{ kg})(2.8\text{ m/s}^2) \\ &= 2720\text{ nts.} \end{aligned}$$

Note: Whenever you can, make the acceleration term *a* a magnitude. That is what I've done above (the acceleration's negative sign has been unembedded). Be careful when you do this, though. Don't put a negative sign in front of the *a*, then proceed to use ***-2.8 m/s²*** when it comes time to put in the numbers.

e.) Moving with a constant velocity means that the acceleration *a* is zero. Friction is still acting (upward in this case), so the f.b.d. used in *Part d* is still valid (the forces haven't changed, there is just no acceleration).

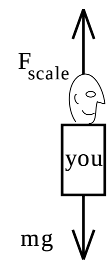
$\underline{\sum F_y}$:

$$\begin{aligned} T - mg + f_k &= -ma \\ \Rightarrow\quad T &= mg - f_k - m(0) \\ &= (400\text{ kg})(9.8\text{m/s}^2) - (80\text{ nt}) \\ &= 3840\text{ nts.} \end{aligned}$$

5.32) The scale in this case is measuring the net force you apply to the scale (or the ground). If the acceleration is upward, this force F_{scale} will be *greater than mg*; if downward, it will be *less than mg*. To determine the acceleration direction, we need to determine *mg*:

$$\begin{aligned} mg &= (60\ \text{kg})(9.8\ \text{m/s}^2) \\ &= 588\ \text{newtons.} \end{aligned}$$

As this is less than the scale reading of 860 newtons, the elevator must be accelerating upward and the acceleration's sign must be *positive.*



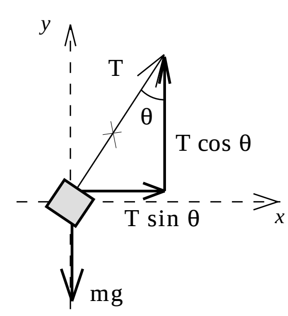
By Newton's Third Law, the force you apply to the scale must be equal and opposite the force the scale applies to you. As such, using an f.b.d. and N.S.L. on yourself (see to right) yields:

$\underline{\sum F_y}$:

$$\begin{aligned} F_{scale} - mg &= ma \\ \Rightarrow \quad a &= (F_{scale}/m) - g \\ &= (860\ \text{nt})/(60\ \text{kg}) - (9.8\text{m/s}^2) \\ &= 4.53\ \text{m/s}^2. \end{aligned}$$

Note: If we had assumed a downward acceleration (i.e., an acceleration that was *negative*), we would have gotten a negative sign in front of the calculated *a* term above. The *negative sign* in an answer like that does not identify direction. By unembedding the sign, we have made the acceleration term a *magnitude.* As such, it should be positive. The negative sign in front of an answer in such instances means we have assumed the *wrong* direction for the acceleration, nothing else!

5.33)



a.) *An f.b.d.* for the forces on the mass is shown to the right. Noting that the acceleration is to the right, I have put one coordinate axis along the horizontal. N.S.L. in the *x direction* yields:

$\underline{\sum F_x}$:

$$\begin{aligned} T \sin\theta &= ma \\ \Rightarrow \quad a &= (T \sin\theta)/m \qquad \text{(Equation A).} \end{aligned}$$

We need to determine T to solve this. Using N.S.L. in the *y direction* yields:

$\underline{\sum F_y:}$

$$T\cos\theta - mg = ma_y$$
$$= 0 \quad (\text{as } a_y = 0)$$

$$\Rightarrow \quad T = mg/(\cos\theta).$$

Re-writing, then substituting back into *Equation A* yields:

$$a = [T](\sin\theta)/m$$
$$= [mg/(\cos\theta)](\sin\theta)/m.$$

The *m's* cancel and $(\sin\theta)/(\cos\theta)$ is $\tan\theta$, so we end up with

$$a = g\tan\theta.$$

For our problem, the numbers yield:

$$a = (9.8\text{ m/s}^2)(\tan 26^o)$$
$$= 4.78\text{ m/s}^2.$$

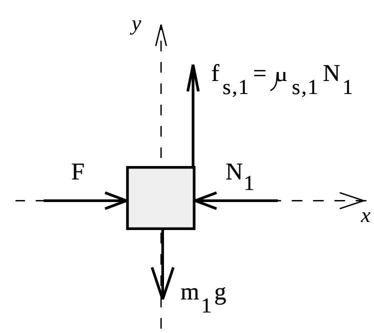
b.) At constant velocity, there is no acceleration and, hence, no swing observed. The string and mass should hang completely vertical. Note: That is exactly what the equation in the *x direction* suggests. The only time the acceleration will equal zero in $T\sin\theta = ma$ is when $\theta = 0$.

Note: One intrepid student whose father was a pilot pointed out that airplane floors (and ceilings) are not horizontal (she observed that when she walks to the bathroom at the rear of a plane, she always walks down hill). In any case, that idiosyncracy isn't important here as the angle is measured relative to the vertical.

5.34)

a.) We are interested in finding the *coefficient of static friction* between both m_1 and m_2 (call this $\mu_{s,1}$) and between m_2 and the wall (call this $\mu_{s,2}$), when F = *25 newtons.*

--To the right is the f.b.d. for m_1. N.S.L. yields:



f.b.d. on mass m_1

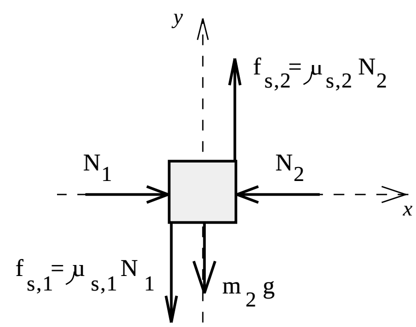
$\underline{\sum F_x}$:

$$F - N_1 = m_1 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad N_1 = F \quad (\text{equal to 25 nts}).$$

$\underline{\sum F_y}$:

$$\mu_{s,1} N_1 - m_1 g = m_1 a_1$$
$$= 0 \qquad (\text{as } a_1 = 0).$$
$$\Rightarrow \quad \mu_{s,1} = (m_1 g)/N_1$$
$$= [(2\text{ kg})(9.8\text{ m/s}^2)] / (25\text{ nt})$$
$$= .784 \qquad (\text{note that the coefficient is unitless}).$$

--The f.b.d. for m_2 is shown to the right. A number of observations need to be made before dealing with N.S.L.:



f.b.d. on mass m_2

i.) Look at m_1*'s* f.b.d. on the previous page. Notice that it experiences a *normal force* N_1 due to its being jammed up against m_2. As such, m_2 must feel a *reaction force* (Newton's Third Law) of the same magnitude (i.e., N_1) in the *opposite direction*. That force has been placed on m_2*'s* f.b.d.

ii.) Look again at m_1*'s* f.b.d. on the previous page. Notice that it experiences a frictional force $f_{s,1}$ due to its rubbing up against m_2. As such, m_2 must feel a *reaction force* of magnitude $f_{s,1}$ in the direction opposite that of the frictional force on m_1. That force has been placed on m_2*'s* f.b.d.

iii.) Having made those observations, N.S.L. yields:

$\underline{\sum F_x}$:

$$N_1 - N_2 = m_2 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad N_1 = N_2 \quad (\text{equal to } F = 25\ nts \text{ as } N_1 = F) .$$

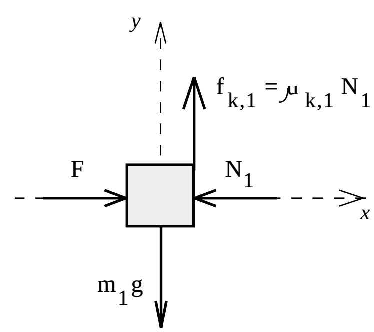
$\underline{\sum F_y}$:

$$\mu_{s,2}N_2 - \mu_{s,1}N_1 - m_2 g = m_2 a_2$$
$$= 0 \quad (\text{as } a_2 = 0)$$
$$\Rightarrow \quad \mu_{s,2} = [\mu_{s,1}N_1 + m_2 g] / N_2$$
$$= [(.784)(25\text{ nt}) + (7\text{ kg})(9.8\text{ m/s}^2)] / (25\text{ nt})$$
$$= 3.528.$$

b.) The force F is now 20 newtons. That means there is not enough force associated with F to keep the bodies pinned to the wall. That being the case, they begin to accelerate downward. Assume the *coefficients of kinetic friction* are $\mu_{k,1} = .15$ and $\mu_{k,2} = .9$ respectively.

As innocuous as this scenario may seem, the problem has the potential to be a real stinker. Why? Because the direction of a frictional force on a body depends upon the direction of its slide *relative to the other body*. We don't know the *acceleration* of each of the bodies. We do know that if m_2 accelerates downward <u>faster</u> <u>than</u> m_1, then m_1*'s* motion *relative to* m_2 will be <u>upward</u> and the frictional force on m_1 will be *downward*. If m_2 accelerates downward <u>more slowly</u> than m_1, then m_1*'s* motion *relative to* m_2 will be <u>downward</u> and the frictional force on m_1 will be *upward*. Not knowing the acceleration of either body means we don't know which body will be moving faster and, hence, what direction the frictional force will be on either object. In short, we have to do the problem both ways to see which ends up making sense.

We will start by assuming m_1 accelerates faster than m_2. In that case, the frictional force on m_1 will be upward and the f.b.d. for the situation will be as shown to the right. Using N.S.L. on m_1, we get:



f.b.d. on mass m_1

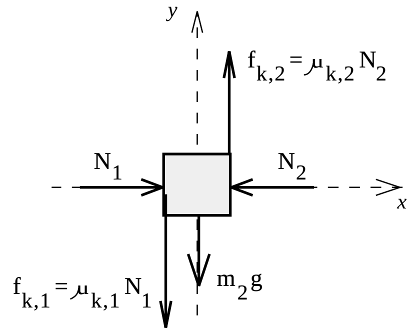
$\underline{\sum F_x}$:

$$F - N_1 = m_1 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad F = N_1 \quad (\text{equal to 20 nt}).$$

$\underline{\sum F_y}$:

$$\mu_{k,1}N_1 - m_1 g = -m_1 a_1.$$

$$\Rightarrow \quad a_1 = [-\mu_{k,1}N_1 + m_1 g]/m_1$$
$$= [-(.15)(20\text{ nt}) + (2\text{ kg})(9.8\text{ m/s}^2)] / (2\text{ kg})$$
$$= 8.3\text{ m/s}^2.$$



f.b.d. on mass m_2

--The f.b.d. for the forces acting on m_2 are shown on the next page. N.S.L. yields:

$\underline{\sum F_x:}$

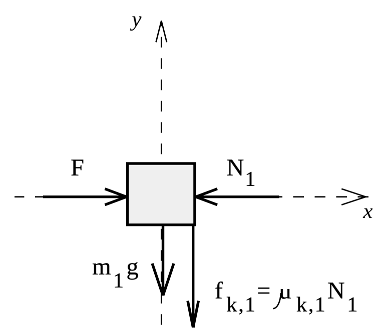
$$N_1 - N_2 = m_2 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad N_1 = N_2 \quad (\text{equal to } F = 20\text{ nts}).$$

$\underline{\sum F_y:}$

$$\mu_{k,2}N_2 - \mu_{k,1}N_1 - m_2 g = -m_2 a_2$$
$$\Rightarrow \quad a_2 = [-\mu_{k,2}N_2 + \mu_{k,1}N_1 + m_2 g] / m_2$$
$$= [-(.9)(20\text{ nt}) + (.15\text{ kg})(20\text{ nt}) + (7\text{ kg})(9.8\text{ m/s}^2)] / (7\text{ kg})$$
$$= 7.66\text{ m/s}^2.$$

Note 1: Yes! We've lucked out. We assumed m_1 accelerates faster than m_2, and that is just what our calculations have verified. If we had been wrong, we would have gotten senseless results. As we got it right on the first try, we needn't go further.

Note 2: For the amusement of it, let's go further. That is, assume that m_1 accelerates *more slowly* than m_2. That means m_1 will slide *upward* relative to m_2 and the frictional force will be downward (this is exactly opposite the situation we outlined above). With the direction of the frictional force reversed, the f.b.d. on m_1 look as shown to the right. N.S.L. yields:



f.b.d. on mass m_1

$\underline{\sum F_x:}$

$$F - N_1 = m_1 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad F = N_1 \quad (= 20\text{ nts}).$$

$\underline{\sum F_y}$:

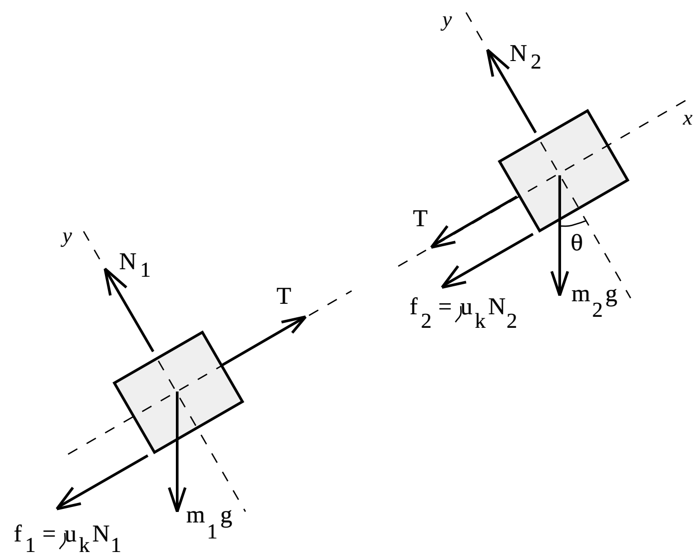
$$-\mu_{k,1}N_1 - m_1g = -m_1a_1.$$
$$\Rightarrow \quad a_1 = [\mu_{k,1}N_1 + m_1g]/m_1$$
$$= [(.15)(20\text{ nt}) + (2\text{ kg})(9.8\text{ m/s}^2)] / (2\text{ kg})$$
$$= 11.3\text{ m/s}^2.$$

Yikes! According to our calculations, block m_1 is accelerating faster than the *acceleration of gravity* ($g = 9.8\ m/s^2$). That isn't possible in this situation. Conclusion? We made bad assumptions about the acceleration of m_1 and m_2.

c.) The reason the accelerations are different? They have different forces acting on them!

5.35)

a.) The *free body diagrams* for this situation are shown to the right.



b.) We need the frictional forces in both cases, which means we need both N_1 and N_2. Using N.S.L. in the *y direction*:

$\underline{\sum F_y}$:

$$N_1 - m_1g\cos\theta = m_1a_y$$
$$\Rightarrow \quad N_1 = m_1g\cos\theta \qquad (\text{as } a_y = 0).$$

Likewise, $N_2 = m_2g\cos\theta$.

--Using N.S.L. for the *x-motion* of m_1, noting that the acceleration is in the *negative* direction, relative to our coordinate axis (the body is slowing, hence the acceleration is *opposite* the direction of the velocity):

$\underline{\sum F_x}$:

$$T - \mu_kN_1 - m_1g\sin\theta = -m_1a.$$

Substituting in for N_1 and solving for $m_1 a$, we get:

$$m_1 a = [-T + \mu_k(m_1 g \cos\theta) + m_1 g \sin\theta] \qquad \text{(Equation A).}$$

--To get rid of the tension term, consider the *x motion* of m_2:

$\underline{\sum F_x}$:

$$-T - \mu_k N_2 - m_2 g \sin\theta = -m_2 a.$$

Substituting in for N_2 and solving for the tension T, we get:

$$T = -\mu_k(m_2 g \cos\theta) - m_2 g \sin\theta + m_2 a.$$

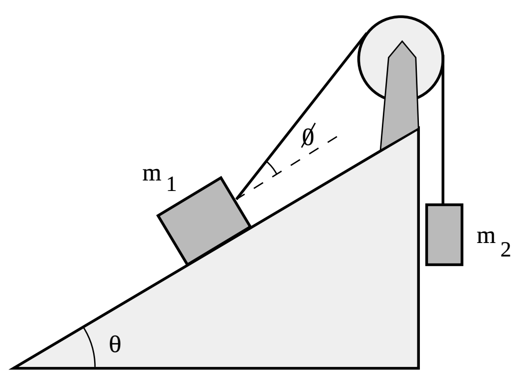
Substituting the tension term into *Equation A* yields:

$$m_1 a = [-(-\mu_k(m_2 g \cos\theta) - m_2 g \sin\theta + m_2 a) + \mu_k(m_1 g \cos\theta) + m_1 g \sin\theta].$$

Solving for the acceleration yields:

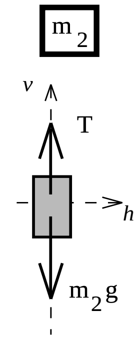
$$a = [\mu_k(m_2 g \cos\theta) + m_2 g \sin\theta + \mu_k(m_1 g \cos\theta) + m_1 g \sin\theta]/(m_1 + m_2)$$
$$= \mu_k g \cos\theta + g \sin\theta.$$

c.) Plugging the expression for a back into *Equation A* allows us to determine T. I'll save space by leaving the exercise to you.



5.36) This is an important situation because it requires you to face all the pitfalls that can occur when doing incline-plane problems.

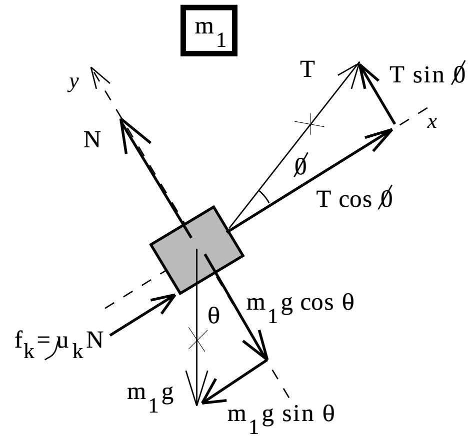
We know m_1 is moving *down* the incline. That means m_2 is moving upward. Unfortunately, we have not been told the direction of *acceleration* for either m_1 or m_2. For the sake of amusement, let's assume m_1*'s* acceleration is *up* the incline (i.e., it's slowing). That will make m_2*'s*



acceleration (remember, it's physically moving upward) *downward* (i.e., it's also slowing). Consider m_2*'s* f.b.d. first. N.S.L. allows us to write:

$\underline{\sum F_y}$:

$$T - m_2g = -m_2a_2$$
$$\Rightarrow \quad T = m_2g - m_2a_2 \qquad \text{(Equation 1).}$$



Remembering that the magnitude of m_1*'s* acceleration is numerically equal to $a_2 \cos\theta$ (this was pointed out in the original set-up), now consider m_1*'s* f.b.d. N.S.L. yields:

$\underline{\sum F_x}$:

$$T\cos\phi + \mu_k N - m_1g\sin\theta = m_1a_1$$
$$\Rightarrow \quad T\cos\phi + \mu_k N - m_1g\sin\theta = m_1(a_2\cos\theta) \qquad \text{(Equation 2).}$$

At this point, we have three unknowns *N*, *a*, and *T*. To determine an expression for *N*, consider N.S.L. in the *y direction* for m_1. Doing so yields:

$\underline{\sum F_y}$:

$$T\sin\phi + N - m_1g\cos\theta = m_1a_y = 0 \qquad (\text{as } a_y = 0)$$
$$\Rightarrow \quad N = -T\sin\phi + m_1g\cos\theta \qquad \text{(Equation 3)}$$

--Note that although the problem did not ask you to do so, solving for a_2 is done in the following manner.

Plugging *Equation 1* into *Equation 3* yields:

$$N = -(m_2g - m_2a_2)\sin\phi + m_1g\cos\theta \qquad \text{(Equation 4)}$$

Plugging *Equation 1* and *Equation 4* into *Equation 2* yields:

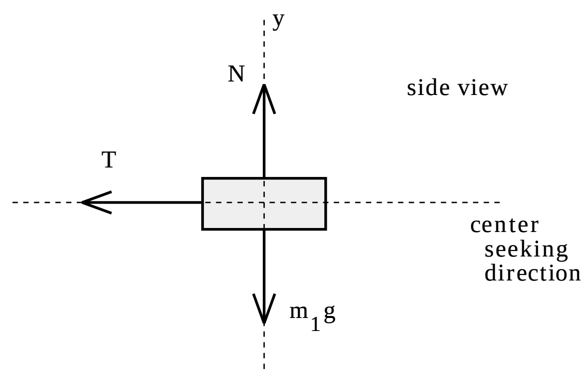
$$\underbrace{(m_2g - m_2a_2)}_{T}\cos\phi + \mu_k\underbrace{[-(m_2g - m_2a_2)\sin\phi + m_1g\cos\theta}_{N} - m_1g\sin\theta = m_1(a_2\cos\phi)$$

Rearranging and solving for a_2 yields:

$$a_2 = \frac{m_2 g\cos\phi - \mu_k m_2 g\sin\phi + \mu_k m_1 g\cos\theta - m_1 g\sin\theta}{m_1\cos\phi + m_2\cos\phi - \mu_k m_2\sin\phi} .$$

Interesting Note: There are positive and negative parts of the denominator, but it's OK because the two amounts will never add to zero.

5.37) This is a circular motion problem. There must be a natural force somewhere in the system that acts to change the direction of *m_1's* motion. That is, there must be a *gravitational* or *normal* or *tension* or *friction* or *push-me-pull-you* force that is *center-seeking*. In this case, that force provided by the system is the tension in the string. The problem proceeds:

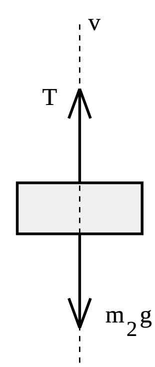


Using N.S.L. on mass m_1 (see f.b.d. to right):

$\underline{\sum F_c}$:

$$-T = -m_1 a_c$$
$$\Rightarrow \quad T = m_1 (v^2/R)$$
$$\Rightarrow \quad v = (TR/m_1)^{1/2}.$$

This equation has two unknowns, v and T. To get rid of the tension term, consider N.S.L. applied to mass m_2 (see f.b.d. to right):



$\underline{\sum F_v}$:

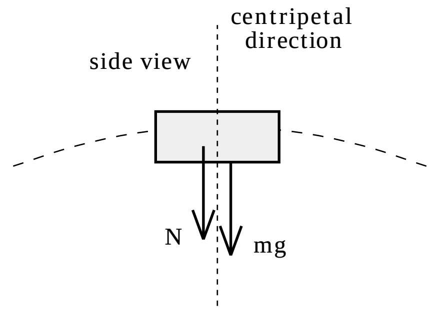
$$T - m_2 g = 0 \qquad (\text{as } a_y = 0)$$
$$\Rightarrow \quad T = m_2 g.$$

Substituting back into our expression for v, we get:

$$v = [TR/m_1]^{1/2}$$
$$= [(m_2 g)R/m_1]^{1/2}.$$

This is a nice problem as it requires you to deal with more than one body. The approach is the same as it has always been. Do *an f.b.d.* for one body in the system. In this case, notice that the body is moving in a circular path. As such, orient one axis so that it is *center-seeking* (i.e., along the radius of the arc upon which the bob is moving). Use N.S.L. to generate as many equations as needed. If you haven't enough equations to solve for the desired unknown, pick a second mass and repeat the approach.

5.38) *An f.b.d.* for the forces acting on the cart when at the top of the loop is shown to the right. N.S.L. yields:



$\underline{\sum F_c}$:

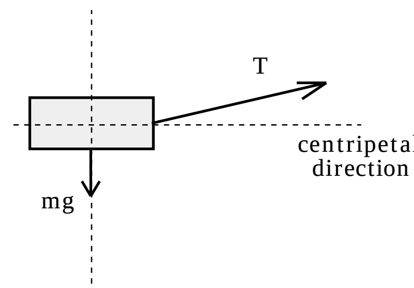
$$\begin{aligned} -N - mg &= -m\,a_c \\ &= -m\,(v^2/R) \\ \Rightarrow \quad v &= [(N + mg)R/m]^{1/2}. \end{aligned}$$

When the cart just freefalls through the top of the arc, the normal force goes to zero. In that case:

$$v = [gR]^{1/2}.$$

5.39)

a.) To begin with, the tension vector must have a *vertical component* (see f.b.d. to the right). If it doesn't, there will be nothing to counteract gravity and the rock must accelerate downward--something our object is not doing. As such, that vertical force will ALWAYS equal *mg*. BUT, if the rock is moving fast, the angle will be small and the vertical component will be very small *in comparison to T*. In that case, we can assume the tension force *T* is *wholly centripetal* and *r = L*. Using those assumptions:



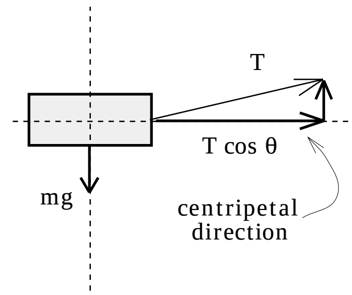
$\underline{\sum F_c}$:

$$\begin{aligned} T &= m\,a_c \\ &= m\,(v^2/L) \\ \Rightarrow \quad v &= [TL/m]^{1/2}. \end{aligned}$$

Putting in the numbers and using T_{max}, this yields:

$$
\begin{aligned}
v &= [TL/m]^{1/2} \\
&= [(50\ nt)(1.2\ m)/(.2\ kg)]^{1/2} \\
&= 17.32\ m/s.
\end{aligned}
$$

b) Because there is centripetal motion going on here, the temptation is to draw *an f.b.d.* like the one shown to the right and then sum the forces in the *center-seeking direction*. Noting that the radius *r* of the body's motion is $L \cos\ \theta$, we write:



$\underline{\sum F_c:}$

$$
\begin{aligned}
T \cos\ \theta &= m\, a_c \\
&= m\, [v^2/r] \\
&= m\, [v^2/(L \cos\ \theta)] \\
\Rightarrow \quad (\cos\ \theta)^2 &= [mv^2/LT].
\end{aligned}
$$

This equation would be great if we knew the velocity and wanted the angle (or vice versa). Unfortunately, we know neither. In other words, for this particular question, summing in the *center-seeking direction* is going to be no help at all (at least not initially). With that in mind, let's use N.S.L. in the *vertical direction* and pray it gives us an equation we can use.

$\underline{\sum F_v:}$

$$
\begin{aligned}
&T \sin\ \theta - mg = 0 \qquad (\text{as } a_y = 0) \\
&\Rightarrow \quad \sin\ \theta = mg / T_{max} \\
&\qquad\qquad = (.2\ kg)(9.8\ m/s^2) / (50\ nt) \\
&\qquad\qquad = .039 \\
&\Rightarrow \quad \theta = 2.247^o.
\end{aligned}
$$

c.) We now know the angle that corresponds to the velocity at which the string will give up and break. With that information we can use N.S.L. in the *center-seeking* direction to bring the velocity term into play (that equation was derived above--it is re-derived below for your convenience). Doing so yields:

$\underline{\sum F_c}$:

$$T \cos \theta = m a_c$$
$$= m [v^2/(L \cos \theta)]$$
$$\Rightarrow v = [LT(\cos \theta)^2/m]^{1/2}$$
$$= [(1.2 \text{ m}) (50 \text{ nt}) (\cos 2.247^o)^2 / (.2 \text{ kg})]^{1/2}$$
$$= 17.3 \text{ m/s}.$$

Notice how close this is to the solution determined in *Part a*. The reason for this should be obvious. The string-breaking velocity is high which means the string-breaking angle is small. Being so, the vertical tension component (this must equal *mg*) will be small in comparison to the overall tension *T* and the *horizontal tension component* will very nearly equal *T*. The assumption we made in *Part a* was that the tension was all in the center-seeking (horizontal) direction--in this case, that wasn't a bad assumption to make.

d.) For this part, we must incorporate the velocity into our analysis (we didn't do that when we were looking for the angle in *Part b*; you should understand the difference between these two situations). Using the f.b.d. shown in *Part b-i*, we can use N.S.L. to write:

$\underline{\sum F_c}$:

$$T \cos \theta = m a_c$$
$$= m [v^2/(L \cos \theta)]$$
$$\Rightarrow v = [TL(\cos \theta)^2/m]^{1/2} \qquad \text{(Equation A).}$$

In this case, we don't know *T*. Looking at the vertical forces yields:

$\underline{\sum F_v}$:

$$T \sin \theta - mg = 0 \qquad (\text{as } a_y = 0)$$
$$\Rightarrow T = mg/\sin \theta.$$

Substituting *T* into Equation A:

$$v = [T(\cos \theta)^2 L/m]^{1/2}$$
$$= [(mg/\sin \theta) (\cos \theta)^2 L/m]^{1/2}$$
$$= [(g/\sin \theta) (\cos \theta)^2 L]^{1/2}$$
$$= [g (\cot \theta) (\cos \theta) L]^{1/2}.$$

NOTE: If you don't like the use of the cotangent function (cos/sin), forget it and simply use the sine and cosine terms as presented.

Putting in the numbers, we get:

$$v = [(9.8\ m/s^2)(\cot 30^o)(\cos 30^o)(1.2\ m)]^{1/2}$$
$$= 4.2\ m/s.$$

5.40)

a.) The gravitational force between you and the earth, using Newton's general gravitational expression, is:

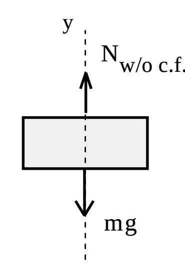
$$F_g = G\ m_{you}m_e/r^2$$
$$= (6.67x10^{-11}\ m^3/kg\cdot s^2)\ (70\ kg)\ (5.98x10^{24}\ kg)\ /\ (6.37x10^6\ m)^2$$
$$= 688\ nts.$$

Using $m_{you}g$:

$$F_g = m_{you}g$$
$$= (70\ kg)\ (9.8\ m/s^2)$$
$$= 686\ nts.$$

The discrepancy is due to round-off error.

Note: The reason we can get away with using *mg* when near the earth's surface is due to the fact that the earth's radius is so large. That is, it really doesn't matter whether you are on the earth's surface or 200 meters above the earth's surface. For all intents and purposes, the distance between you and the center of the earth is going to be, to a very good approximation, the same.



b.) Let's begin by determining the amount of normal force ($N_{w/o\ c.f.}$) the earth must apply to you *when you stand at the poles*. The f.b.d. for the situation is shown to the right. Noting that there is no centripetal acceleration at the poles (at the poles the rotational speed of the earth is zero), a_y is zero and N.S.L. yields:

$\underline{\sum F_y:}$

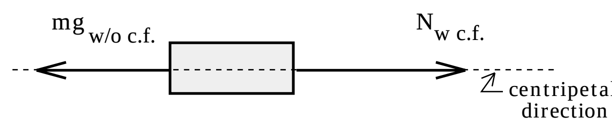
$$N_{w/o\ c.f.} - mg_{w/o\ c.f.} = 0 \qquad (\text{as } a_y = 0)$$

$$\Rightarrow \quad N_{w/o\ c.f.} = mg_{w/o\ c.f.}$$
$$= (70\text{ kg})(9.83\text{ m/s}^2)$$
$$= 688.1\text{ newtons.}$$

Note that at the equator, the earth's rotational speed is equal to *the distance a point on the equator travels in one day* (i.e., the circumference = $2\pi R = (2)(3.14)(6.37\text{x}10^6\text{ m}) = 4\text{x}10^7\text{ m}$) divided by *the time it takes to do the traveling* (i.e., *24 hours = 86,400 seconds*), or:

$$v_{eq} = d / t$$
$$= (4\text{x}10^7\text{ m}) / (86{,}400\text{ sec})$$
$$= 463.2\text{ m/s} \qquad \text{(this is around 1000 mph).}$$

Let's now determine the amount of normal force ($N_{w\ c.f.}$) the earth must apply to you *when you stand at the equator*. The f.b.d. for the situation is shown to the right. Noting that as there is centripetal acceleration at the equator (at the equator there is rotational speed in the amount calculated above), a_y is non-zero and N.S.L. yields:



$\underline{\sum F_c}$:

$$N_{w\ c.f.} - mg_{w/o\ c.f.} = -ma_c$$
$$= -m(v^2/R)$$
$$\Rightarrow \quad N_{w\ c.f.} = mg_{w/o\ c.f.} - m(v^2/R).$$

Put in a different context, the normal force required at the equator will be equal to the normal force required without centripetal force (remember, $N_{w/o\ c.f.} = mg_{w/o\ c.f.}$ from above) minus the centripetal force (this will numerically equal mv^2/R) required to move you into circular motion. Putting in the numbers yields:

$$N_{w\ c.f.} = mg_{w/o\ c.f.} - m(v^2/R)$$
$$= (688.1\text{ nts}) - (70\text{ kg})[(463.2\text{ m/s})^2/(6.37\text{x}10^6\text{ m})$$
$$= 685.7\text{ nts.}$$

If we wanted to define a gravitational constant g_{equ} that, when multiplied by your mass gives the amount of force the earth must exert on you when you stand at the equator (that is exactly how the *g* value you have come to know and love was originally determined), g_{equ} will be:

$$N_{w\ c.f.} = mg_{w\ c.f.}$$
$$\Rightarrow \quad g_{w\ c.f} = N_{w\ c.f.}/m$$
$$= (685.7 \text{ nts}) / (70 \text{ kg})$$
$$= 9.796 \text{ m/s}^2.$$

c.) Defining the distance between the earth and moon to be *r* and using N.S.L., we get:

$\underline{\sum F_c}$:

$$-G\, m_e m_m/r^2 = -m_m\, a_c$$
$$= -m_m\, v^2/r$$
$$\Rightarrow \quad v = (Gm_e/r)^{1/2} \qquad \text{(Equ. A)}$$
$$= [(6.67\text{x}10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)\ (5.98\text{x}10^{24} \text{ kg}) / (3.84\text{x}10^{8} \text{ m})]^{1/2}$$
$$= 1019 \text{ m/s}.$$

Interesting Note: Just as two objects will accelerate at the same rate (assuming neither gets close to its terminal velocity) under the influence of gravity, the velocity required to pull a mass in a given-radius circular path does NOT depend upon the mass of the object being so motivated. This might not be immediately obvious (just as the first statement in this NOTE wasn't obvious back when you first ran into it), but it is supported by the math. The moon is the mass being centripetally accelerated, and the m_m terms do cancel out in our velocity equation as derived above.